$$= 4\left[(9x^2+4-12x)2+6(2x-3)(3x-2)\right]+6\left[3(4x^2+9-12x)+4(3x-2)(2x-3)\right]$$

$$= 4\left[18x^2-24x+8+36x^2-78x+36\right]+6\left[12x^2+27-72x+24x^2-52x+24\right]$$

$$= 4\left[54x^2-102x+44\right]+6\left[36x^2-124x+51\right]$$

$$= 216x^2-408x+176+216x^2-744x+306$$

$$= 432x^2-1152x+482.$$

when $x = 2/3$

$$f''(2/3) = 192-768+482 = -94 < 0.$$

$$f''(3/2) = 972-1728+482 = -274 < 0.$$

$$f''(13/12) = 507-1248+482 = -259 < 0.$$

f'' is minimum at $x = 2/3, 3/2, 13/12$.

$$f(x) = (3x-2)^2(2x-3)^2$$

$$f(2/3) = \left(3\times \cancel{2/3}-2\right)^2\left(2\times 2/3-3\right)^2$$

$$= 0.$$

$$f(3/2) = \left(3\times 3/2-2\right)^2\left(2\times \cancel{3/2}-3\right)^2 = 0.$$

$$f(13/12) = \left(\frac{3\times 13}{12}-2\right)^2\left(\frac{2\times 13}{12}-3\right)^2$$

$$= \left(\frac{39-24}{12}\right)^2\left(\frac{26-36}{12}\right)^2$$

Necessary condition	Sufficient condition	Nature of fn	conclusion
$f'(x_0)=0$	$f'(x_0)=f''(x_0)=\dots=f^{n-1}(x_0)=0$ and $f^n(x_0)<0$, n is even	concave	Local maximum at x_0.
$f'(x_0)=0$	$f'(x_0)=f''(x_0)=\dots=f^{n-1}(x_0)=0$ and $f^n(x_0)>0$, n is even	convex.	Local minimum at $x=x_0$
$f'(x_0)=0$	$f'(x_0)=f''(x_0)=\dots=f^{n-1}(x_0)=0$ and $f^n(x_0)\neq 0$ n is odd.	—	Point of inflection at $x=x_0$ or Saddle point at $x=x_0$.

note:

Remarks:

1. A local minimum of a convex function on a convex set is also a global minimum of that function.

2. A local maximum of a concave function on a convex set is also a global maximum

3. A global ~~local~~ minimum of a strictly convex function on a convex set is also a unique global minimum of that function.

4. A local maximum of a strictly concave function on a convex set is also a unique global maximum of that function.

optimisation multi variable functions.

Let $f(x) = f(x_1, x_2, \ldots x_n)$. we define the gradient vector of $f(x)$ denoted by $\nabla f(x)$ and defined by $\nabla f(x) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots \frac{\partial f}{\partial x_n}\right)$ and Hessian matrix denoted by $H(x)$ defined by $H(x)=$

$$H(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \frac{\partial^2 f}{\partial x_n \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

Note 1:

$H(x)$ is positive definite if all its leading in this case the stationary point is a local minimum. A principle minor of $H(x)$ is a determinant of square submatrix whose elem lie on the diagonal of $H(x)$ whereas leading principle minor whose (1, 1) element is the (1, 1) element of $H(x)$.

note 2: $H(x)$ is negative definite. If the signs of all even ~~principle~~ leading principle minors are positive and or if the principle minor of $H(x)$ are altern in sign.

3. If signs of determinants do not meet condi 1 and 2, then the stationary point may be either a maximum or minimum or neither in this case the matrix $H(x)$ is termed a semi definite or indefinite.

Example:

consider $H(x)$ as $\begin{bmatrix} 5 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -2 & 4 \end{bmatrix}$ the

leading principle minors of this matrix

$$\begin{vmatrix} 5 & 3 \\ 3 & 4 \end{vmatrix} = 20 - 9 = 11.$$

$$\begin{vmatrix} 5 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -2 & 4 \end{vmatrix} = 5(16-2) - 3(12-0) + 0$$

$$= 5(14) - \cancel{B}3(12)$$

$$= 70 - 36$$

$$= 34.$$

Necessary condition	Sufficient condition	Conclusion
$\nabla f(x_0) = 0$	$H(x_0)$ is positive definite	Local minimum at $x = x_0$
$\nabla f(x_0) = 0$	$H(x_0)$ is negative definite and if.	Local ~~minimum~~ maximum at $x = x_0$
$\nabla f(x_0) = 0$	$H(x_0)$ is semidefinite or indefinite.	Point of inflection at $x = x_0$.

Problems:

1. Consider the function $f(x) = x_1 + 2x_2 + x_1x_2 - x_1^2 - x_2^2$. Determine the maximum or minimum (if any) of the function.

$$\nabla f(x) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2} \right).$$

Hessian matrix $H(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \end{bmatrix}$

Diff ① p.w.r.t x_1

$$\frac{\partial f}{\partial x_1} = 1+x_2-2x_1 \quad , \quad \frac{\partial^2 f}{\partial x_1^2} = -2.$$

Diff ① p.w.r.t x_2

$$\frac{\partial f}{\partial x_2} = 2+x_1-2x_2 \quad , \quad \frac{\partial^2 f}{\partial x_2^2} = -2.$$

$$\nabla f(x) = (1+x_2-2x_1 \, , \, 2+x_1-2x_2).$$

$$\nabla f(x) = 0$$

$1+x_2-2x_1 = 0 \to$ ② and $2+x_1-2x_2 = 0 \to$ ③

Solving ② and ③.

$-2x_1 + x_2 = -1 \to$ ②

$x_1 - 2x_2 = -2 \to$ ③

② × 2 ⇒ $-4x_1 + 2x_2 = -2$

③ ⇒ $x_1 - 2x_2 = -2$

$-3x_1 = -4.$

$$\boxed{x_1 = 4/3}$$

Sub $x = 4/3$ in ②

$$-\frac{8}{3} + x_2 = -1$$

$$x_2 = -1 + 8/3$$

$$\boxed{x_2 = 5/3}$$

$\therefore x_0 = (x_1, x_2)$

$x_0 = (4/3, 5/3)$

$\nabla f(x_0) = 0$ at $x_1 = 4/3$ and $x_2 = 5/3$.

$\frac{\partial^2 f}{\partial x_2 \partial x_1} = 1 \qquad \frac{\partial^2 f}{\partial x_1 \partial x_2} = 1.$

$$H(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$H(x) = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}.$$

leading principle minors are $|-2| = -2$

$$\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 4-1 = 3.$$

Therefore the leading principle minors are alternatingin signs. $\therefore H(x)$ is negative definite

$\Rightarrow$ at $x = x_0$ ie $= (4/3, 5/3)$ given function is maximum $\nabla f(x) = 0$.

$\therefore$ The maximum value of the function is

$x_1 = 4/3 \quad x_2 = 5/3.$

$\therefore f(x) = \frac{4}{3} + 2(5/3) + (4/3)(5/3) - (4/3)^2 - (5/3)^2$

$= 4/3 + \frac{10}{3} + \frac{20}{9} - \frac{16}{9} - \frac{25}{9}$

90
12
1012
-16
86
-25
61
38
1
62
-16
46
-21
21

2. Examine the following function for extreme points $f(x_1, x_2) = 3x_1^2 + x_2^2 - 10$

$$\nabla f(x) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2} \right)$$

$$H(x) = \begin{vmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 F}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{vmatrix}$$

Diff $f(x_1, x_2)$ partially w.r.t x_1

$$\frac{\partial f}{\partial x_1} = 6x_1 \qquad \frac{\partial^2 f}{\partial x_2 \partial x_1} = 0. \qquad \frac{\partial^2 f}{\partial x_1^2} = 6.$$

Diff $f(x_1, x_2)$ partially w.r.t x_2

$$\frac{\partial f}{\partial x_2} = 2x_2 \qquad \frac{\partial^2 f}{\partial x_1 \partial x_2} = 0. \qquad \frac{\partial^2 f}{\partial x_2^2} = 2$$

$$\nabla f(x) = (6x_1, 2x_2)$$

$$\nabla f(x) = 0$$

$$6x_1 = 0 \qquad 2x_2 = 0$$

$$\boxed{x_1 = 0} \qquad \boxed{x_2 = 0}$$

$$x_0 = (0, 0)$$

$\nabla f(x_0) = 0$ when $x_1 = 0$, $x_2 = 0$.

$$H(x) = \begin{vmatrix} 6 & 0 \\ 0 & 2 \end{vmatrix} = 12.$$

$$\begin{vmatrix} 6 & 0 \\ 0 & 2 \end{vmatrix} = 12.$$

$H(x)$ is positive definite. $\Rightarrow$ at $x=0$ i.e $(0,0)$.

given function is minimum $\nabla f(x)=0$.

$\therefore$ The minimum value of the function is

$x_1=0, \; x_2=0.$

$f(x) = -10.$

ii) $f(x) = 3(0) + (0)^2 - 10$

$= 3\times 0 + 0 - 10$

$= 0 - 10$

$$\boxed{f(x) = -10}$$

5. Determine the maximum and minimum for the.

fn $f(x) = f(x_1, x_2 \ldots x_n) = x_1^2 + 2x_2^2 + x_3^2 + x_1x_2 - 2x_3$

$- 7x_1 + 12.$

$$f(x_1, x_2, x_3) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3}\right)$$

$$H(x) = \begin{vmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{vmatrix}$$

$$\frac{\partial f}{\partial x_1} = 2x_1 + x_2 - 7, \quad \frac{\partial^2 f}{\partial x_1 \partial x_2} = 1, \quad \frac{\partial^2 f}{\partial x_1 \partial x_3} = 0$$

$$\frac{\partial f}{\partial x_2} = 4x_2 + x_1, \quad \frac{\partial^2 f}{\partial x_2 \partial x_1} = 1, \quad \frac{\partial^2 f}{\partial x_2 x_3} = 0.$$

$$\frac{\partial f}{\partial x_3} = 2x_3 - 2, \quad \frac{\partial^2 f}{\partial x_3 \partial x_1} = 0, \quad \frac{\partial^2 f}{\partial x_3 \partial x_2} = 0.$$

$$\frac{\partial^2 f}{\partial x_1^2} = 2, \quad \frac{\partial^2 f}{\partial x_2^2} = 4. \quad \frac{\partial^2 f}{\partial x_3^2} = 2.$$

$$\nabla f(x) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3}\right)$$

$$= (2x_1 + x_2 - 7,\ 4x_2 + x_1,\ 2x_3 - 2)$$

$\nabla f(x) = 0,$

$2x_1 + x_2 = 7 \rightarrow (1)$

$x_1 + 4x_2 = 0 \rightarrow (2)$

$2x_3 = 2. \Rightarrow \boxed{x_3 = 1}$

$(1) \times 4 \Rightarrow 8x_1 + 4x_2 = 28$

$(2) \Rightarrow x_1 + 4x_2 = 0$

$\quad (-) \quad (-) \quad (-)$

$7x_1 = 28.$

$\boxed{x_1 = 4}$ Sub $x_1 = 4$ in (1).

$x_2 = 7 - 8$

$\boxed{x_2 = -1}$

$$H(x) = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

Leading principle minors are $|2| = 2$.

$$\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} = 8 - 1 = 7.$$

$$\begin{vmatrix} 2 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 2(8) - 1(2) = 16 - 2 = 14.$$

$H(x)$ is positive definite. at $x = x_0$ ie $(4, -1, 1)$.

given function is minimum at $\nabla f(x) = 0$.

The minimum value of the function

$$= (4)^2 + 2(-1)^2 + (4)(-1) - 2(1) - 7(4) + 12 + 1$$

$$= 16 + \cancel{2} - 4 - \cancel{2} - 28 + 12 + 1$$

$$= -3.$$

Constrained Multivariable optimisation with equality constraints.

Method I :

La

Method I: This method is applicable for only if the problem contains only one equality constraints.

Problem:

1. Find the optimum solution of the following constrained multivariable problem.

minimize $Z = x_1^2 + (x_2+1)^2 + (x_3-1)^2 \rightarrow (1)$.

Sub to c/s: $x_1 + \cancel{5}x_2 - 3x_3 = 6$.

Step: Eliminate any one of the variable in the function z by using given constraint

Consider $x_1 + 5x_2 - 3x_3 = 6$.

$$-3x_3 = 6 - x_1 - 5x_2$$

$$\boxed{x_3 = \frac{x_1 + 5x_2 - 6}{3}}$$

Sub this x_3 value in (1)

$$Z = (x_1)^2 + (x_2+1)^2 + \left(\frac{x_1+5x_2-6}{3} - 1\right)^2$$

$$= x_1^2 + (x_2+1)^2 + \left(\frac{x_1+5x_2-9}{3}\right)^2.$$

The above function χ is optimisation without constraint form,

$$\nabla \chi(x) = \left(\frac{\partial \chi}{\partial x_1}, \frac{\partial \chi}{\partial x_2} \right).$$

$$H(x) = \begin{vmatrix} \frac{\partial^2 \chi}{\partial x_1^2} & \frac{\partial^2 \chi}{\partial x_1 \partial x_2} \\ \frac{\partial^2 \chi}{\partial x_2 \partial x_1} & \frac{\partial^2 \chi}{\partial x_2^2} \end{vmatrix}$$

$$\chi = x_1^2 + (x_2+1)^2 + \frac{(x_1+5x_2-9)^2}{9}.$$

Diff p.w.r.t x_1 and x_2

$$\frac{\partial \chi}{\partial x_1} = 2x_1 + \frac{2}{9}\cancel{(1)}\ \frac{2}{9}(x_1+5x_2-9)(1) = \frac{8x_1 + 10x_2 - 18}{3} \qquad \frac{20x_1+10x_2-18}{9}$$

$$\frac{\partial \chi}{\partial x_2} = 2(x_2+1) + \frac{2}{9}(x_1+5x_2-9)(+5)$$

$$= 2x_2 + 2 + \frac{10x_1 + 50x_2 \mp 90}{9}$$

$$= \frac{18x_2 + 18 + 10x_1 + 50x_2 \mp 90}{9}$$

$$= \frac{68x_2 + 10x_1 + \cancel{108} - 72}{9}$$

$$= \frac{10x_1 + 68x_2 - 72}{9}.$$

$$\frac{20x_1+10x_2-18}{9}=0 \qquad \frac{10x_1+68x_2-72}{9}=0.$$

$20x_1+10x_2=18 \rightarrow (1).$

$10x_1+68x_2=72 \rightarrow (2).$

$(1)\times 1 \Rightarrow 20x_1+10x_2=18$

$(2)\times 2 \Rightarrow 20x_1+136x_2=144$

$\quad (-) \quad (-) \quad (-)$

$-126x_2=-126.$

$\boxed{x_2=1}$

Sub $x_2=1$ in (1).

$20x_1+10=18.$

$20x_1=18-10$

$20x_1=8.$

$\boxed{x_1=4/10}$, $\boxed{x_1=2/5}$

$$H(x)=\begin{vmatrix} \frac{20}{9} & \frac{10}{9} \\ \frac{10}{9} & \frac{68}{9} \end{vmatrix}$$

Leading principle minors $\left|\frac{20}{9}\right|=\frac{20}{9}$

$$\begin{vmatrix} \frac{20}{9} & \frac{10}{9} \\ \frac{10}{9} & \frac{68}{9} \end{vmatrix} = \frac{1360}{9^2}-\frac{100}{9^2}$$

$= \frac{1260}{\ldots} \ldots 126\ldots$

H(x) is positive definite at $x = x_0$ ie $\left(\frac{2}{5}, 1\right)$.

given sub $x_1 = \frac{2}{5}$ and $x_2 = 1$ in

$$x_3 = \frac{x_1 + 5x_2 - 6}{3}$$

$$= \frac{2/5 + 5 - 6}{3} = \frac{2/5 - 1}{3}$$

$$= \frac{-3/5}{3} = -\frac{1}{5}$$

$$\boxed{x_3 = -\frac{1}{5}}$$

minimum value of the function is:

$$= \left(\frac{2}{5}\right)^2 + (1+1)^2 + \left(-\frac{1}{5} - 1\right)^2$$

$$= \frac{4}{25} + 4 + \left(-\frac{6}{5}\right)^2$$

$$= \frac{4}{25} + 4 + \frac{36}{25} = \frac{4+36+100}{25} = \frac{140}{25}$$

$$= \frac{28}{5}$$

$\therefore$ The minimum value of the function $= \frac{28}{5}$

2. optimize $z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3 \rightarrow ①$

Sub to cts: $x_1 + x_2 + x_3 = 7$.

Eliminate any one of the variable in the function z by using given constraint.

Consider $x_1+x_2+x_3=7$

$x_3 = 7 - x_1 - x_2$.

Sub this value in (1)

$$z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$$

$$= x_1^2 - 10x_1 + x_2^2 - 6x_2 + (7-x_1-x_2)^2 - 4(7-x_1-x_2)$$

$$= x_1^2 - 10x_1 + x_2^2 - 6x_2 + (7-x_1-x_2)^2 - 28 + 4x_1 + 4x_2$$

$$= x_1^2 + x_2^2 - 6x_1 - 2x_2 - 28 + (7-x_1-x_2)^2$$

$$= x_1^2 + x_2^2 - 6x_1 - 2x_2 + 28 + (49 + x_1^2 + x_2^2 - 14x_1 + 2x_1x_2 - 14x_2$$

$$= x_1^2 + x_2^2 - 6x_1 - 2x_2 + 28 + 49 + x_1^2 + x_2^2 - 14x_1 + 2x_1x_2 - 14x_2$$

$$= 2x_1^2 + 2x_2^2 - 20x_1 - 16x_2 + 2x_1x_2 + 77$$

$$\nabla z(x) = \left(\frac{\partial z}{\partial x_1}, \frac{\partial z}{\partial x_2}\right)$$

$$H(x) = \begin{vmatrix} \frac{\partial^2 z}{\partial x_1^2} & \frac{\partial^2 z}{\partial x_1 \partial x_2} \\ \frac{\partial^2 z}{} & \frac{\partial^2 z}{} \end{vmatrix}$$

Diff partially with respect to x_1

$$\frac{\partial x}{\partial x_1} = 4x_1 \cancel{/-20x_1}\ 4x_1 - 20 + 2x_2.$$

$$\frac{\partial x}{\partial x_2} = 4x_2 - 16 + 2x_1$$

$$\nabla x = 0.$$

$$(4x_1 - 20 + 2x_2,\ 4x_2 - 16 + 2x_1) = 0.$$

$$4x_1 + 2x_2 = 20 \rightarrow (1)$$

$$2x_1 + 4x_2 = 16. \rightarrow (2).$$

Solve (1) and (2) $\quad (1) \times 2 \Rightarrow 8x_1 + 4x_2 = 40$

$$(2) \Rightarrow 2x_1 + 4x_2 = 16$$

$$(-) \quad (-) \quad (-)$$

$$6x_1 = 24 \qquad 40 - 16 = 24$$

$$\boxed{x_1 = 4}$$

Sub $x_1 = 4$, in $4(4) + 2x_2 = 20$

$$16 + 2x_2 = 20$$

$$2x_2 = 20 - 16$$

$$2x_2 = 4$$

$$\boxed{x_2 = 2}$$

$$\frac{\partial^2 f}{\partial x_1^2} = 4. \quad \frac{\partial^2 f}{\partial x_2^2} = 4.$$

$$\frac{\partial^2 f}{} = 2. \quad \frac{\partial^2 f}{} = 2.$$

$$H(x) = \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix}$$

The leading minors are $|4| = 4$.

$$\begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} = 16 - 4 = 12.$$

$H(x)$ is a positive definite at $x = x_0$ ie $(4, 2)$

Find x_3 value.

$$x_1 + x_2 + x_3 = 7$$

$$4 + 2 + x_3 = 7, \quad x_3 = 7 - 6$$

$$\boxed{x_3 = 1}$$

minimum value of the function is, $(4, 2$

$$z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$$

$$= (4)^2 - 10(4) + (2)^2 - 6(2) + (1)^2 - 4(1).$$

$$= 16 - 40 + \cancel{4} - 12 + 1 - \cancel{4}$$

$$\boxed{z = -35}$$

Lagrange multiply method: CNLP with constraints equality more important

~~Sufficient~~ Necessary condition for general problem:

consider the problem optimize $z = f(x)$

Sub to cts: $h_i(x) = b_i$ are

(or)

$g_i(x) = h_i(x) - b_i$, $i = 1, 2, \ldots m$.

and $m \leq n$.

Here m is number of constraints, n is number of variables.

Let the Lagrangian function for a general non-linear programming problem involving n variables and m constraints be

$$L(x, \lambda) = f(x) - \sum_{i=1}^{m} \lambda_i g_i(x).$$

Further the necessary conditions $\frac{\partial L}{\partial x_j} = 0$,

$j = 1, 2, \ldots n$.

$$\frac{\partial L}{\partial \lambda_i} = 0, \; i = 1, 2, \ldots m.$$

for an extreme point to be local optimum of $f(x)$ is also true for optimum of $L(x, \lambda)$. Let the ... point points x and λ satisfying the

$$\nabla L(x,\lambda) = \nabla f(x) - \sum_{i=1}^{m} \lambda_i g_i(x) = 0.$$

and $g_i(x) = 0 \quad i = 1, 2, \dots n.$

Necessary and sufficient condition then the sufficient condition for an extreme point be local minimum (or local maximum) of $f(x)$ subject to the constraints $g_i(x) = 0$ $i = 1$ is the determinante of the matrix (called Bordered Hessian matrix). $D = \begin{bmatrix} Q & H^T \\ H & O \end{bmatrix}_{(m+n)\times(m+n)}$

$Q = \left[\dfrac{\partial^2 L}{\partial x_i \partial x_j}\right]_{n\times n}$ $H = \left[\dfrac{\partial g_i(x)}{\partial x_j}\right]_{m\times n}.$

The sufficient condition for the maxima and minima is determined by the ~~[illegible]~~ signs the last $(n-m)$ principle minors of Matr

i) 1. If starting with principle minor of order $m+1$. The extreme point x gives

values of the objective functi

when science signs when of last $(n-m)$ principle minors alternate in signs starting with -1^{m+n} sign.

2. If starting with principle minor of order $2m+1$ the extreme point x gives the minimum value of the objective function, when all signs of $n-m$ principle minors are of same ~~f~~ sign and of -1^{m} sign.

Problem:

1. Solve the following problem by using the method of Lagrangial multiply method.

Minimize $z = x_1^2 + x_2^2 + x_3^2$.

Sub to c/s: $x_1 + x_2 + 3x_3 = 2$.

$5x_1 + 2x_2 + x_3 = 5$

Soln:

minimize $z = x_1^2 + x_2^2 + x_3^2$.

Sub to c/s: $g_1(x) = x_1 + x_2 + 3x_3 - 2 = 0 \rightarrow (1)$.

$h_1(x)$

$g_2(x) = 5x_1 + 2x_2 + x_3 - 5 = 0 \rightarrow (2)$.

$h_2(x)$

$\boxed{m = 2.}$

$L(x, \lambda) = f(x) - \sum^{2} \lambda_i g_i(x)$.